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THE "RATIO" CHART FOR PLOTTING STATISTICS.

BY PROFESSOR IRVING FISHER, *Yale University*.

COMPARING TWO MAGNITUDES BY THEIR DIFFERENCE AND BY THEIR RATIO.

In the last few years there has been a great increase in the appreciation and use of statistical charts. As a consequence, a number of efforts have been made to improve the technique of graphic representation in order that the chart may convey its message more quickly, accurately, and unmistakably. Toward this end Mr. Willard C. Brinton in particular has put us all under obligation.*

In the present article, I shall describe the nature and advantages of one form of chart on which to plot a statistical curve,—by which is meant any curve or broken line showing the different numerical values of a statistical magnitude at different periods of time. This chart will be found of very great assistance when the only object or the chief object is to display and compare *ratios*. It may therefore be called the "ratio" chart.†

The chief reason why ratio charting has not yet been more widely used is, I believe, that its extreme simplicity is not yet realized. Those who have mentioned it in print have usually contented themselves with stating dogmatically how it is to be used without explaining the whys and wherefores, further than that its spacing is "logarithmic" like that of a logarithmic slide rule. But most persons regard logarithms and slide rules as a species of magic and fight shy of a method, the foundations of which they do not clearly understand.

*Through the publication of his "Graphic Methods for Presenting Facts," New York (The Engineering Magazine Company), 1914, 371 pp., and through the work of the Joint Committee on Standards for Graphic Representation which grew out of the publication of his book.

†I am not, as I originally supposed, the first to hit on this simple device, although it still remains almost wholly unknown and unused and has never yet, so far as I know, been adequately described. A bibliography of the very meagre literature on the subject is given at the end of this article. It is there shown that the method in essence was used as early as 1863.

The object of this article is to make the construction of the ratio chart clear to any reader and, at the same time, to point out in some detail its various uses and advantages. In doing this it is quite unnecessary to use even the concept of logarithms.

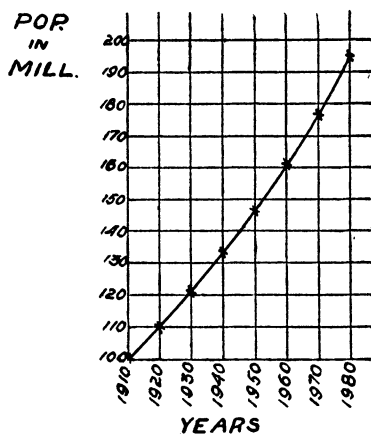


FIG. 1d. FUTURE HYPOTHETICAL POPULATION OF THE UNITED STATES. OR. ORDINARY OR DIFFERENCE METHOD.

Equal vertical intervals represent equal statistical differences. A line in ascending at a uniform ratio is curved. Uniformity is therefore not evident to the eye.

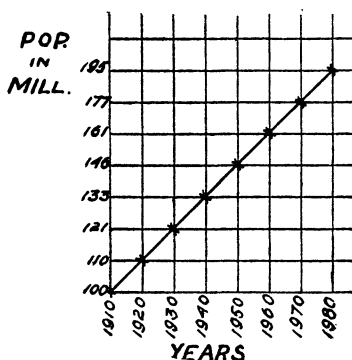


FIG. 1r. THE SAME. RATIO METHOD.

The same assumption. Uniformity is here evident to the eye, being represented by a straight line.

We may compare any two magnitudes, of like kind, by means either of their difference or of their ratio. In the first kind of comparison "an inch on the end of one's nose" is exactly as much as an inch added to the height of the Washington Monument; in the second kind of comparison, on the other hand, "an inch on the end of one's nose" is an addition of about 40 per cent., or as much as 220 feet added to the height of the Monument.

The ordinary chart is adapted to *difference* comparisons rather than to *ratio* comparisons, whereas the statistician is usually concerned with ratio comparisons far more than with difference comparisons.

CONVERTING ORDINARY PLOTTING PAPER.

Fig. 1 d ("d" standing for "difference") shows an ordinary plotting chart; that is, a chart with equal vertical spacing, labelled for population in millions, the labelling

being for equal statistical *differences*, and each interval of verti-

cal ascent representing an increase of 10 millions over the preceding.

Since the key-idea of the ratio chart is that equal vertical intervals represent equal *ratios* of increase instead of equal differences of increase, it may also be constructed from ordinary cross ruled plotting paper. This may be done in two steps, (1) labelling the existing lines and (2) interpolating new lines. The first step is to label the existing (equidistant) horizontal lines with numbers increasing in a given ratio.

Fig. 1 *r* ("r" standing for "ratio"), shows a ratio chart made from ordinary ruled paper simply by labelling the horizontal lines for *ratios*, each interval of vertical ascent now representing an increase of 10 *per cent.* over the preceding. That is, while the first chart is labelled vertically, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, etc. (each number being 10 more than the one below) the second chart is labelled vertically, 100, 110, 121, 133, 146, 161, 177, 195, etc.* (each number being 10 *per cent.* more than the one below).

The ratio method consists simply in plotting any statistical curve by using the labels† of Fig. 1 *r* instead of those of Fig. 1 *d*.

Thus, let us plot, say, the (imaginary) population of the United States in millions, beginning with 100 millions in 1910 and assuming an increase of 10 per cent. every decade. Since all increases of 10 per cent. are represented in Fig. 1 *r* by equal vertical distances, it is clear that the curve representing population, *i. e.*, connecting the series of points in Fig. 1 *r* lying diagonally on the cross lines, will be a straight line.

Thus we see the first merit of the ratio method of charting the growth of a statistical magnitude: *uniformity* in the percentage rate of growth is pictured by *straightness* in the plotted line.

On the other hand, in the ordinary or difference chart, such as that of Fig. 1 *d*, the same population growth will be represented by an "exponential curve" and its uniformity of per-

*If we carry the numbers backward, that below 100 will be 91 (for 91:100::10:11); that next below 91 will be 83; and so on, indefinitely, down to any number, however small, except zero. Evidently we can approach as near to zero as we please but zero itself recedes into a bottomless pit; there is no base or zero line.

†Of course in the ratio chart as in the difference chart the numbering may be magnified or reduced in any ratio. Thus instead of the numbers 100, 110, 121, etc., we may substitute 1.00, 1.10, 1.21 or .100, .110, .121, etc., etc.

centage rate of growth is altogether lost to the eye. In fact the uninitiated are apt to be misled and to falsely infer from

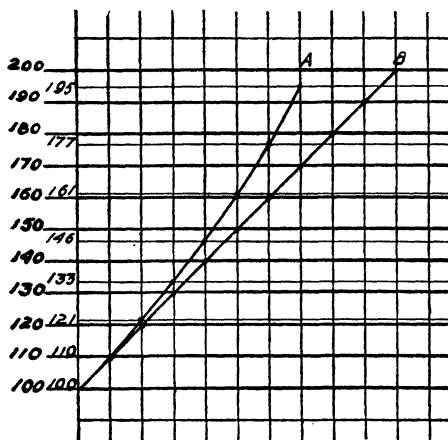


FIG. 2d. SHOWING GEOMETRICAL (A) AND ARITHMETICAL (B) PROGRESSIONS. DIFFERENCE METHOD.

A line (A) ascending at a uniform ratio in equal periods of time, that is, in geometrical progression, is curved (upward). A line (B) ascending by equal differences in equal periods of time, that is, in arithmetical progression, is straight. Note that vertical intervals increasing in geometrical progression (see the light figures) are unequally spaced.

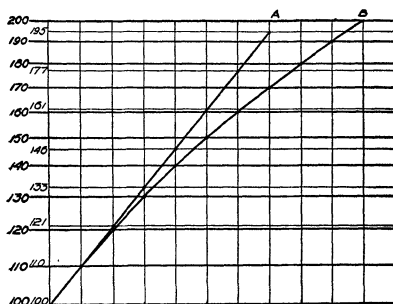


FIG. 2r. THE SAME. RATIO METHOD.

A line (A) ascending at a uniform ratio in equal periods of time, that is, in geometrical progression, is straight. A line (B) ascending by equal differences in equal periods of time, that is, in arithmetical progression, is curved (downward). Note that the vertical intervals increasing in geometrical progression (see the light figures) are equally spaced.

such a curve that the rate of growth is increasing.

RERULING THE RATIO CHART SO OBTAINED.

But the ratio chart, as described above, is not very convenient for plotting points between the ruled lines, because so few of the labels are round numbers. To make a full fledged ratio chart, new horizontal lines need to be interpolated, at their proper places, corresponding to the round numbers 120, 130, 140, 150, etc. (Fig. 2r) and the original equi-distant ones should be erased.* The contrast, then, be-

*Plotting paper for ratio charts may now be obtained ready-made from a few commercial firms, the Educational Exhibition Company, 26 Custom House Street, Providence, R. I., John Wenzel, 63 West 107th Street, New York City, Keuffel and Esser Company, 127 Fulton Street, New York City, and the Standard Graph Co., 32 Union Square, New York City.

The unequally spaced horizontal lines shown in Fig. 2r become more and more crowded together as we ascend, until it becomes necessary, or advisable, to omit some of them.

The omission of these lines causes the plotting paper to present the appearance of an intermittent or cyclical spacing (Fig. 3r).

tween the ratio chart, and the ordinary, or difference, chart, is simply one of spacing, the ratio chart (as in Fig. 2 *r*) having the numbers 120, 130, 140, etc. *unequally* spaced whereas the difference chart (as in Fig. 2 *d*) has these same numbers *equally* spaced.

The above description of a method of forming a ratio chart

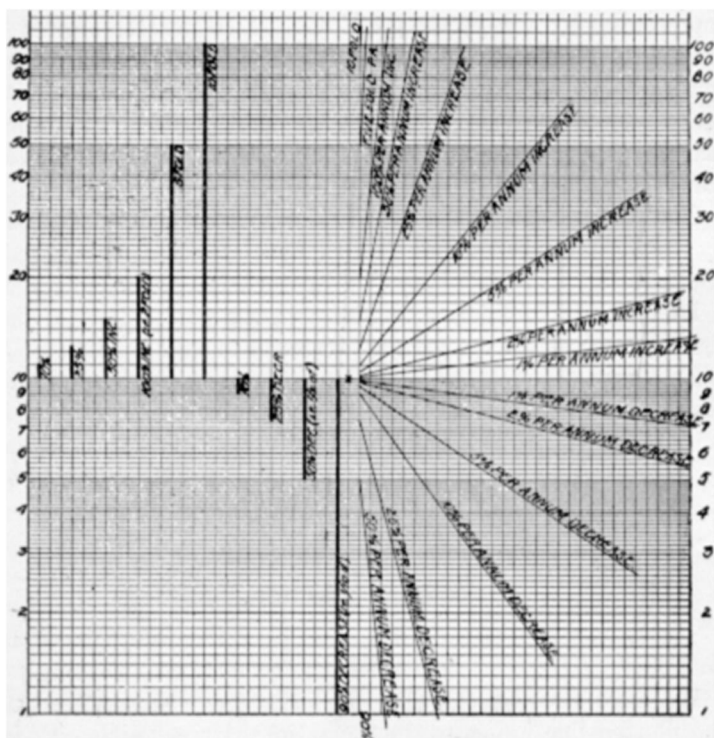


FIG. 3r. SCALES OF ELEVATIONS AND SLOPES. RATIO METHOD.

Showing, at the left, the elevations (and depressions), representing various ratios of increase (or decrease), and showing, at the right, the slopes representing various per annum rates of increase (or decrease).

will serve to explain its nature as representing equal successive percentages of increase by equal intervals on the chart.

In Fig. 3 *r* we see the full fledged ratio chart, constituting a ruling arrangement convenient for plotting. To familiarize the reader with the graphic representation of ratios, some dark vertical lines are drawn. These show how far up to go to

represent an increase respectively of 10, 25, 50, and 100 per cent. (or 2-fold), 5-fold and 10-fold; while the next lines to the right show how far down to go to represent a decrease of 10, 25, 50 (i.e. "half of"), and 90 per cent.. Any two points on the chart, however far removed from each other horizontally, if the vertical interval between them is equal to the 10 per cent. line, will be such that the statistical magnitude represented by the upper point will be 10 per cent. greater than that represented by the lower.. Again the line representing "100 per cent. increase or 2 fold" is the vertical distance between any two points on the chart of which the upper stands for twice the statistical number for which the lower stands.

The reader is advised to verify these and the other legends by taking any two points at random, reading the figures opposite them in the margin and comparing these figures.

He may also familiarize himself with the sloping lines at the right ascending respectively at the rates of 1, 2, 5, 10, 25, 50, 100 per cent. five fold, and ten fold per annum, as well as those descending at the rates of 1, 2, 5, 10, 25, 50, and 90 per cent. per annum, all drawn radiating from a center through the appropriate points on a vertical line one unit to the right of that center.

REPRESENTING GROWTH AT A CONSTANT, OR NEARLY CONSTANT, RATIO.

The advantages of the ratio chart over the difference chart are many. They may be seen from a few illustrative examples.

We have already seen (Fig. 1 *r*) that mere straightness of the plotted line indicates, in the ratio chart, a uniformity in the percentage rate of growth, whereas in the difference chart such uniformity is represented by an exponential curve (Fig. 1 *d*).

In Fig. 2 *d* and Fig. 2 *r* the same lines are repeated and labelled *A*. The line *A* represents a uniform percentage rate of growth. It is an exponential curve in the difference chart (Fig. 2 *d*) and a straight line in the ratio chart (Fig. 2 *r*).

For contrast a straight line *B* is drawn on the difference chart (Fig. 2 *d*). Its straightness signifies little—merely that the same absolute difference is added each year. But the

same absolute difference is a decreasing percentage *rate* of growth and this fact is clearly interpreted in the ratio chart (Fig. 2 *r*). We may say that the chief or typical contrast between the two charts is that a straight line represents in the one an arithmetical progression and in the other a geometrical progression; which is only another way of saying that it

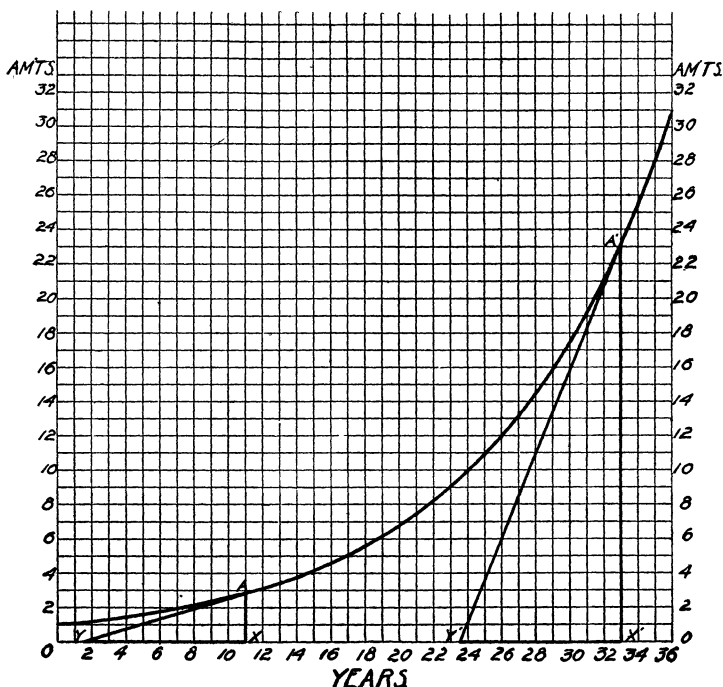


FIG. 4d. UNIFORM PERCENTAGE RATE. DIFFERENCE METHOD.

Showing uselessness of curve at extreme ends; also showing how to compare percentage slopes at different points by comparing (inversely) the subtangents for these points. See p. 596.

represents a progression by equal differences in the one and by equal ratios in the other.

A serious fault in the difference method is that, in a curve of rapid growth, the difference chart is useful only in the middle portion. At the extreme left such a curve, *e. g.*, an exponential, or uniform percentage, curve (as in Fig. 4 *d*) becomes almost indistinguishable from a horizontal line and at the extreme right it becomes almost indistinguishable from a vertical line.

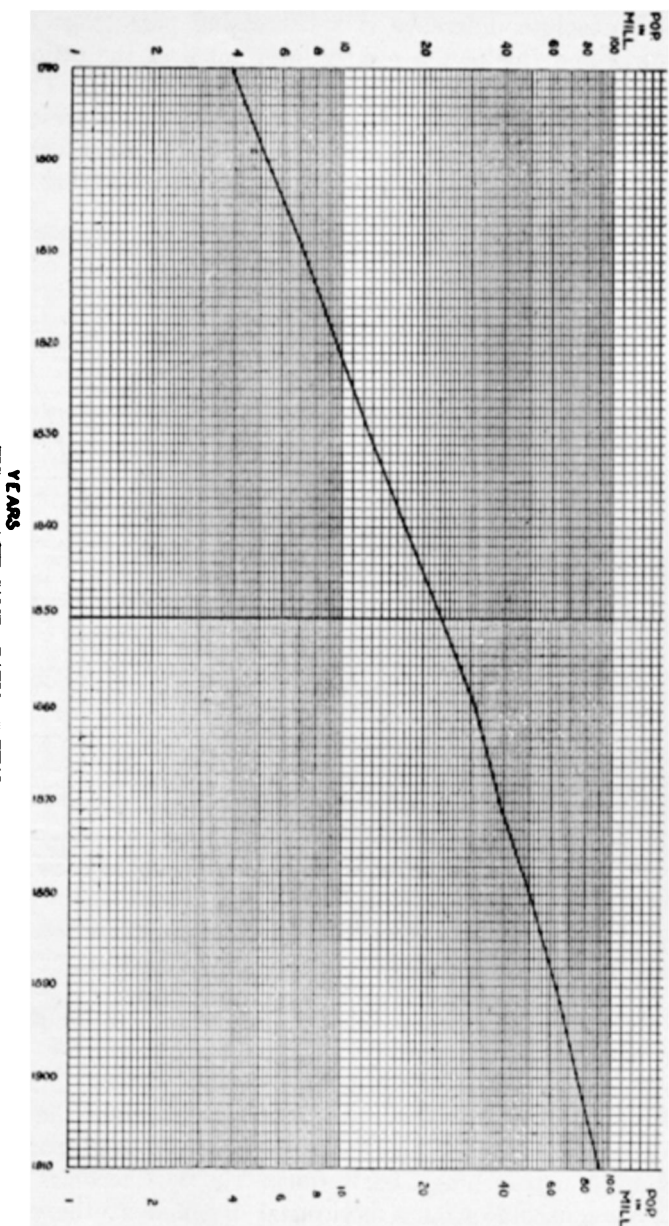
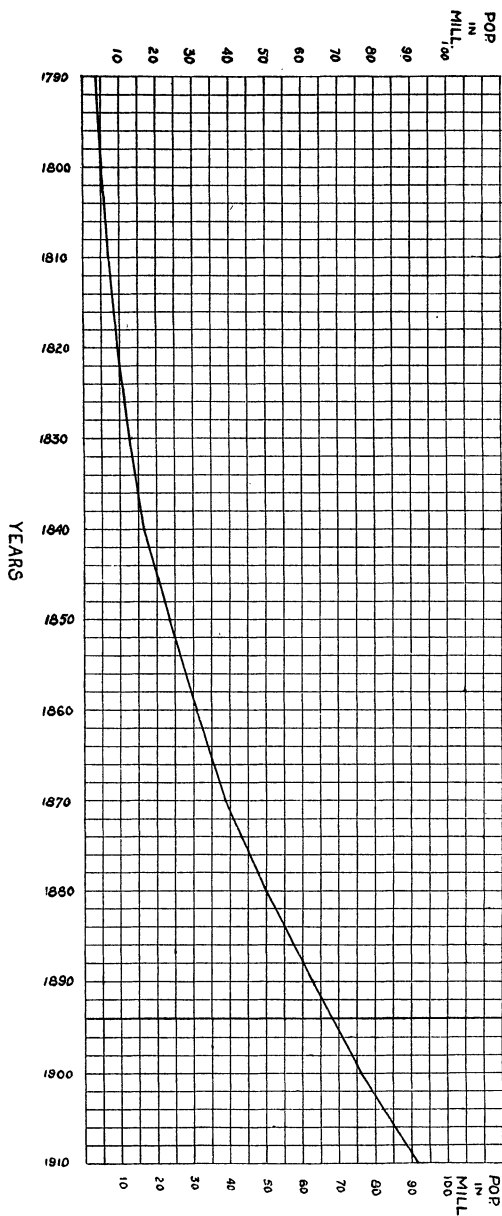


FIG. 5r. THE SAME. RATIO METHOD.
Showing clearly the slight deviations, since 1860, from a uniform rate of growth.



At either extreme no eye can estimate the percentage rate of growth although that rate may not be different from the rate in the middle.

A vast number of statistical charts represent *rapid and long continued growth*,—the statistics of a prosperous business plotted from the beginning; the statistics of a growing country; the statistics of new inventions. We have merely to mention any such familiar examples as statistics of population, wealth, crops, mining, manufacturing, railway mileage, telephones, automobiles, bank deposits, new building, sales of stocks, war debts and other magnitudes rapidly increasing since the war began, in order to realize the thousands or rather, probably, the millions of statistical charts which have been constructed of this kind, most of which are nearly useless at either end.

Thus Fig. 5 *d* represents the actual growth of the population of the United States. The reader, however discerning or experienced, cannot discover by mere ocular inspection of this curve whether the increase at the end is faster or slower than at the beginning. Fig. 5 *r*, on the other hand, shows the facts desired at a glance. It shows a uniform rate of growth between 1790 and 1860 and slight but evident changes since the last named date.

One great advantage of the ratio chart is in forecasting. Usually, in business, we forecast by assuming a certain ratio of growth. In the ratio chart we have simply to draw a straight line; usually, in fact, merely to produce the one already drawn representing the rate experienced in the past.

One method by which users of the ordinary chart have attempted to meet its shortcomings in representing statistics of rapid growth is first to draw a "growth axis," or exponential curve [such as that in Fig. 1 *d*, Fig. 2 *d* (curve A), or Fig. 4 *d*] ascending at the *average* rate of growth of the statistics under consideration; then calculate the per cents. of deviation each year from this "growth axis"; and finally, plot these deviations on a separate chart.

This procedure, however, involves, in addition to charting the original statistical figures, much subsidiary calculation and charting; and the results, when obtained, are not as exact as the results obtainable more easily by the ratio method.

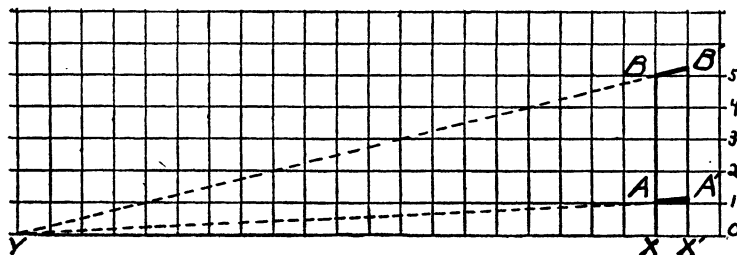


FIG. 6d. EQUAL RATES OF GROWTH APPARENTLY UNEQUAL. DIFFERENCE METHOD.
Segments of two curves ascending at equal rates but having unequal slopes and therefore deceiving the eye.

COMPARING CURVES.

If we wish to compare the growth of population in two countries of different sizes, such as Canada and the United States, the ordinary or difference plots, of which we shall assume that AA' and BB' in Fig. 6 *d* are small sections for a given year, will give the impression that the upper curve, *i. e.*, that for the larger country, is ascending at a much faster ratio than the lower, *i. e.*, that for the smaller country. Few would suspect that the two lines AA' and BB' represent precisely the same percentage rate of growth.

In Fig. 6 *r*, on the other hand, the equality of these same two rates of growth is clearly indicated by the parallelism of the two lines AA' and BB' .

Again, if the plots of the two populations appear as in Fig. 7 *d*, most persons, observing the parallelism of the two lines, would jump to the conclusion that the two populations are growing at the same percentage rate. The fact is that the lower line, AA' , is ascending at a greater percentage rate than the upper, BB' . The ratio chart (Fig. 7 *r*) shows this at a glance.

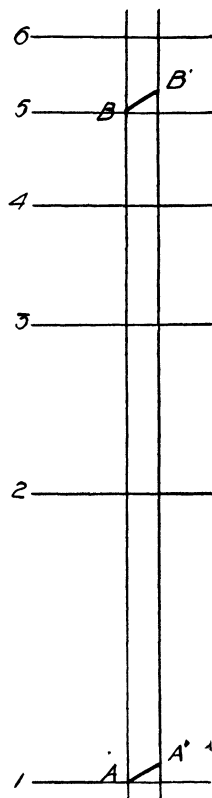


FIG. 6r. EQUAL RATES OF GROWTH EVIDENTLY EQUAL. RATIO METHOD.
The two segments are here parallel.

In the difference chart, in order to make even a rough eye-estimate of the comparative percentage rates of growth of AA' or BB' (Fig. 6 d or Fig. 7 d), we must: (1) note the position of

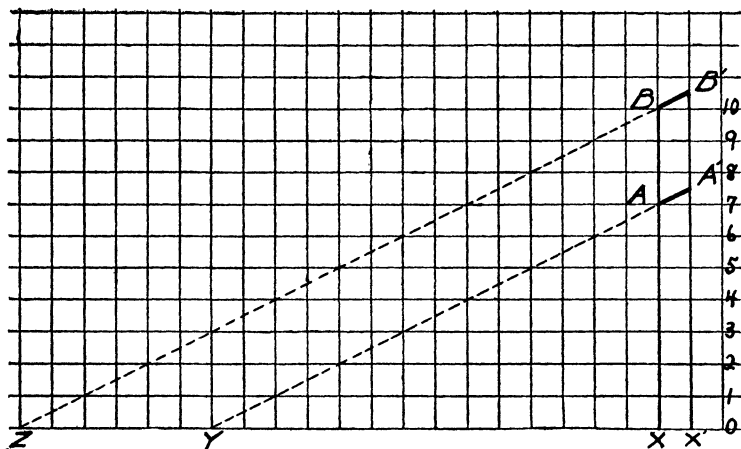


FIG. 7d. UNEQUAL RATES OF GROWTH APPARENTLY EQUAL. DIFFERENCE METHOD.

Segments of two curves ascending at equal slopes, but at unequal rates and therefore deceiving the eye.

the zero or base-line, XX' ; (2) mentally compare $A'X'$ with AX and $B'X'$ with BX ; and (3) compare the two comparisons.

Such mental operations are difficult, irksome, and inaccurate, especially if, as is not uncommon, the base or zero line has been omitted in order to economize space. Furthermore they involve shutting our eyes to the slope or steepness of the lines AA' and BB' , the very feature which first attracts attention.

Often, in fact, the bottom part of the chart, containing the base line, is cut off, and sometimes, instead of using the same base line for two curves on the same chart the draughtsman will bring one curve nearer the other by using two separate base lines. In either event the result is misleading or confusing and in such cases it is almost hopeless to obtain any clear idea

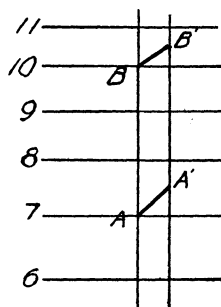


FIG. 7r. UNEQUAL RATES OF GROWTH EVIDENTLY UNEQUAL. RATIO METHOD.

The two segments are here of different slopes and the steeper slope indicates the greater rate of growth.

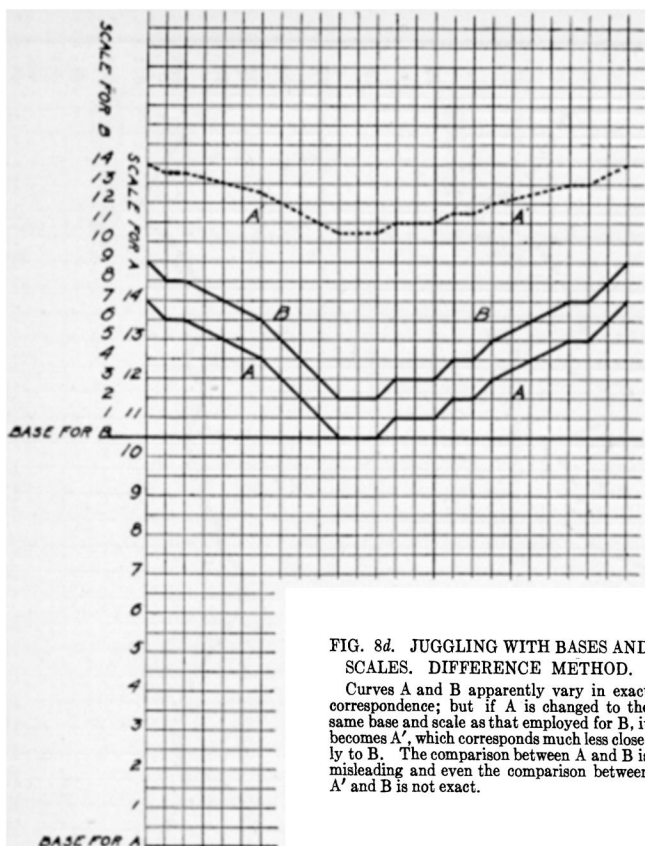


FIG. 8d. JUGGLING WITH BASES AND SCALES. DIFFERENCE METHOD.

Curves A and B apparently vary in exact correspondence; but if A is changed to the same base and scale as that employed for B, it becomes A', which corresponds much less closely to B. The comparison between A and B is misleading and even the comparison between A' and B is not exact.

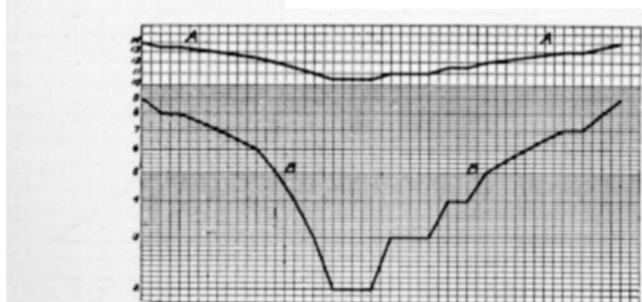


FIG. 8e. THE SAME. RATIO METHOD.

The comparison between A and B here exactly represents the facts, showing only a faint resemblance, in marked contrast with the first comparison in the preceding figure.

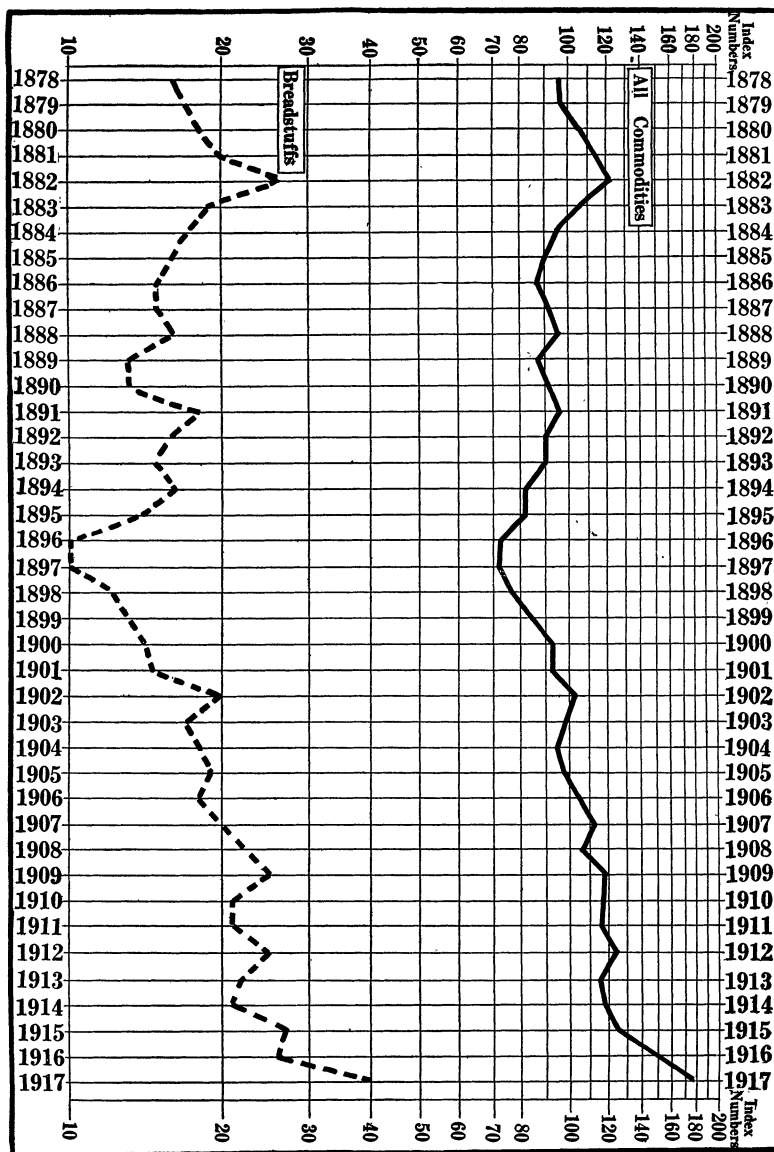


FIG. 97. THE SAME. RATIO METHOD.

Actually "breadstuffs" fluctuate more than "all commodities."

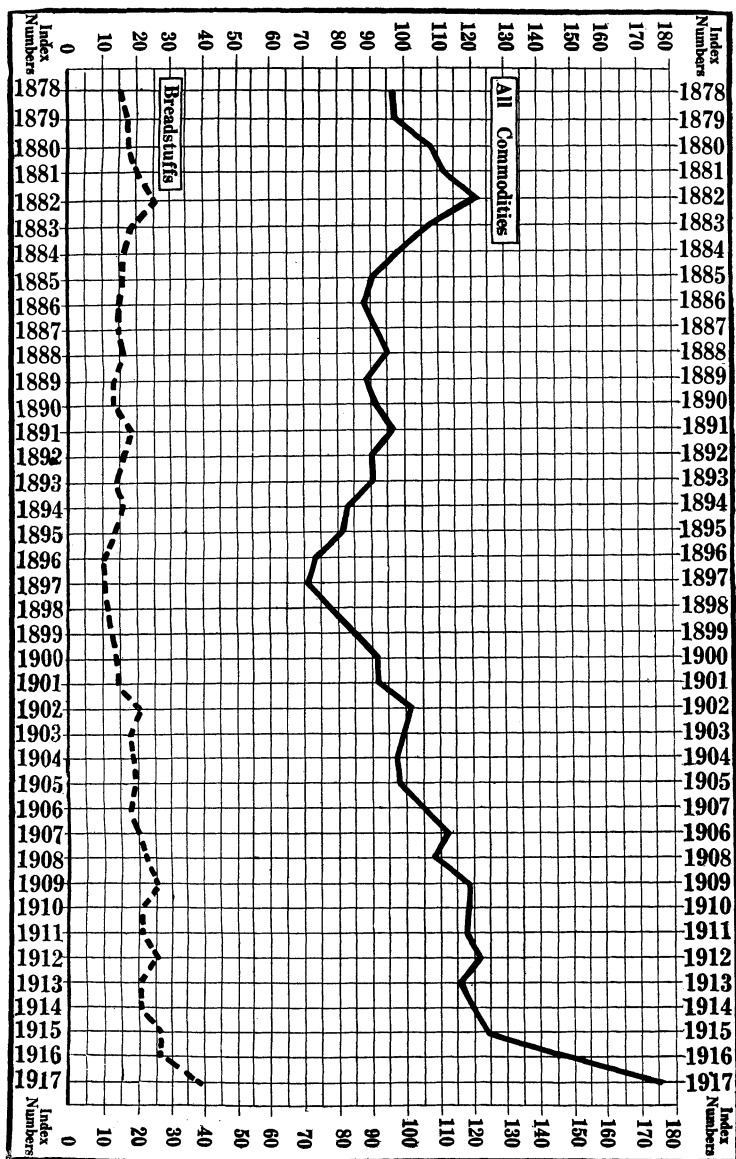


FIG. 9d. INDEX NUMBERS COMPARED. DIFFERENCE METHOD.
Apparently "all commodities" fluctuated more than "breadstuffs."

of the comparative percentage growths except by recourse to tedious arithmetical computations. Thus in Fig. 8 *d* the curves *A* and *B* seem to be exactly similar. But they are far less similar than they appear; for the curve *A* is relative to a remoter base than the curve *B*. If plotted on the same base and scale as *B*, the curve *A* becomes *A'* and its similarity to *B* is greatly diminished. But even this degree of similarity is greater than the statistics warrant, as we see when using the ratio chart (Fig. 8 *r*). This shows the *exact* degree of similarity of the two curves *A* and *B* which turns out to be very small. Thus the ratio chart is an effective means of avoiding juggling with statistics through base-selection or scale-selection.

Fig. 9 *d* and Fig. 9 *r*, taken from a brief article of mine in the *New York Times Annalist* of March 17, 1917, show a type of error opposite to the foregoing, and one taken from actual statistics. Fig. 9 *d* gives the impression that the prices of breadstuffs have fluctuated less than the prices of "all commodities." On the ratio chart, however, as shown in Fig. 9 *r*, although exactly the same numbers are plotted, it is seen that breadstuffs have actually fluctuated a trifle more violently than "all commodities."

For comparing in detail any two curves on ratio charts, such as those in Fig. 9 *r*, we may do what we have just seen is not properly permissible on a difference chart,—we may move bodily either curve, the upper curve downward or the lower curve upward, until the two are close together. Then the various degrees of parallelism or divergence, at various periods of time, may be seen with the utmost clearness. This is done in Fig. 10 *r*.^{*} Such close comparison will usually give quickly, through the eye, a better practical picture, I think, of the degree of correlation and certainly of the location of the correlation, than can be obtained even by laborious calculations of coefficients of correlation.

Index numbers with widely different bases, such as those of Sauerbeck and of the United States Bureau of Labor, if plotted on *difference* charts without especially selecting the scales or without multiplying or dividing by suitable constants, appear

^{*}The scale on the right applies to the lower curve, that on the left to the upper.

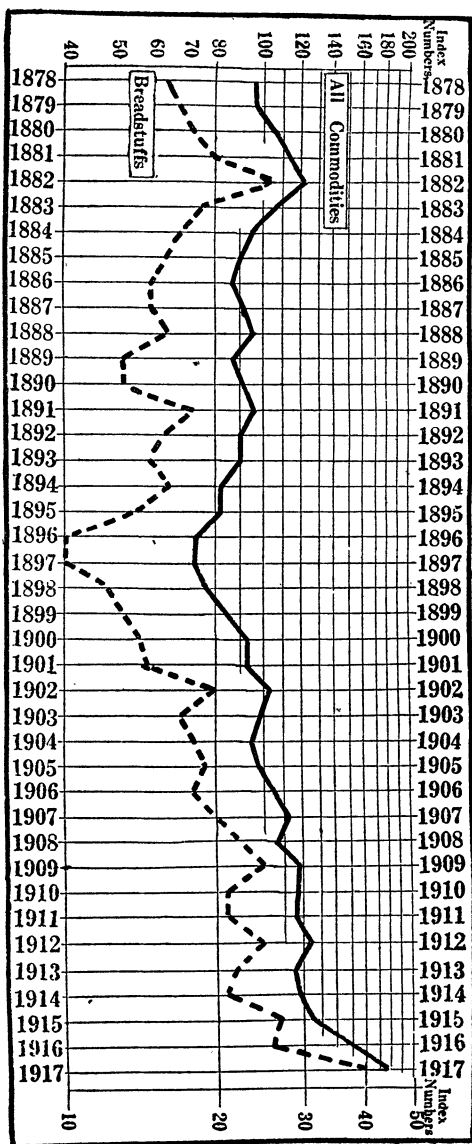


FIG. 10r. THE SAME AS FIG. 9r. RATIO METHOD.
Showing more sharply the contrast by bringing the curves closer together. This procedure is legitimate in the ratio method, but not in the difference method.

far more different from each other than they really are. *Ratio* plotting will show their *exact* similarities and differences.

As was said at the outset, difference charts are useful when difference comparisons are wanted. Not only are such cases rare, however, but often difference comparisons between curves are meaningless, and ratio comparisons are alone possible. This is the case when two curves represent incommensurable magnitudes as, for instance, when a curve of railway gross receipts is compared with one of population or one of the circulation of money with one of the price level. In such cases difference plotting involves a very dangerous discretion in selecting the scale and the base in order neither to exaggerate nor to understate the degree of correspondence. For instance, the close similarity in the recent changes in the money in circulation in the United States, on the one hand, and the price level, on the other, was not evident to me until I plotted the two curves on a ratio chart (Fig. 11 *r*). This similarity might be almost overlooked on a difference chart if the scales for dollars and index numbers were selected arbitrarily.

One special, though slight, advantage of the ratio chart is in representing more clearly than in the difference chart the results of multiplying or dividing a statistical series of numbers by a constant, or by another series, and in exhibiting the resultant third series relatively to the other two.*

ATTEMPTS TO MEND THE FAULTS OF THE DIFFERENCE CHART.

One attempt to secure better comparability of curves in ordinary plotting is that customary in handling index numbers, —namely to reduce all prices to percentages of the base-period price, so that we may start all the curves at an even 100 per

*In detail: (1) to multiply or divide by a constant any series of numbers plotted in a curve is merely to shift the curve bodily up or down by the appropriate constant distance; (2) the reciprocals (of a statistical series plotted in a curve) form a second curve symmetrical (relatively to the unity line) to the first curve; in other words two curves representing reciprocal numbers are such as to coincide if the chart is folded over the unity line; thus a plot representing an index number of prices will be the exact reverse of that representing the purchasing power of money, the one ascending whenever the other descends and at the same angle; (3) the curve formed by plotting the products of two series will be such that its distances from the unity line will be the sum of the differences of the other two from that line; (4) analogously, a quotient of two curves is distant from the unity line the difference of their distances (i. e., distance of the dividend curve less that of the divisor curve). Thus if we plot a curve of acres per *capita* of wheat lands and another of bushels per acre and then plot the curve of their products, i. e., of bushels per capita, the three curves will be seen to be related as just described; the same would apply to index numbers of prices, of wages, and of purchasing power of wages ("real wages").

cent. If the curves do not greatly diverge this gives approximately correct results. But it is a makeshift method and never gives the absolutely exact comparisons of the ratio

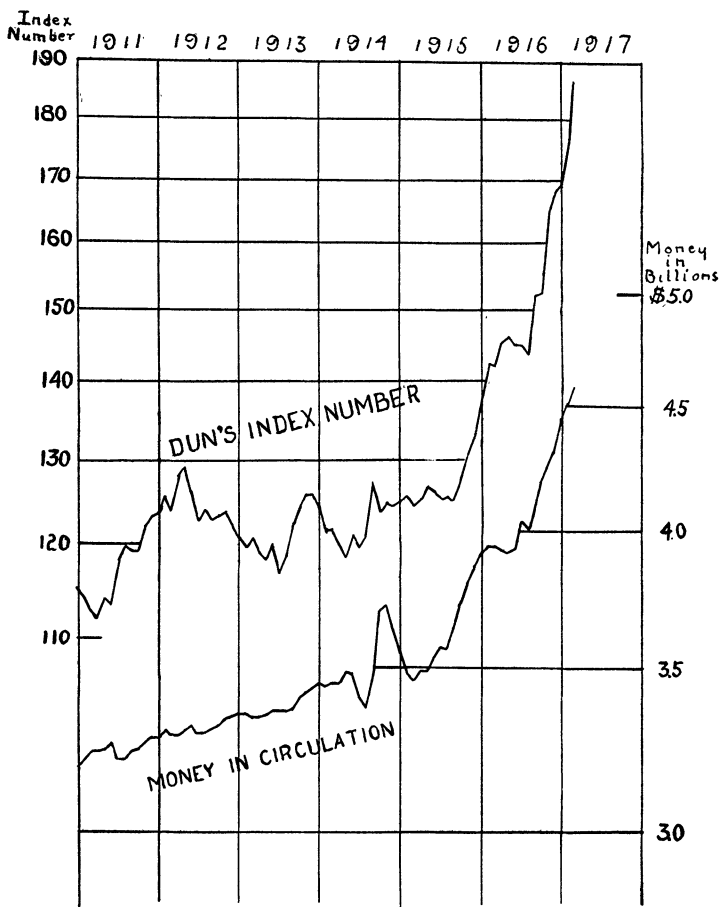


FIG. 11r. MONEY AND THE PRICE LEVEL. RATIO METHOD.

Showing the exact degree in which the price level in the United States has fluctuated in comparison with the amount of money in circulation. Since the war, there has been a close correspondence, changes in the price level following changes in money by two or three months.

method; and, after the curves have diverged considerably, their correct comparison becomes difficult.

Another method is that suggested by Professor Alfred

Marshall. It consists in making a special geometric construction.* But recourse to geometric construction is slow and inexact and does not appeal to the eye.

As a matter of fact, users of charts, when seeking percentage comparisons, are apt simply to read off the four numerical figures opposite the various points (as A and A' and B and B' in Fig. 6 *d* or Fig. 7 *d*) and compute arithmetically the two percentage rates of increase, (*i. e.*, the percentage excess of height of A' over A and that of B' over B). But such a procedure is nothing less than giving up the use of the diagram as a diagram and using it merely as a table of arithmetical figures. A diagram is supposed to interpret figures to the eye and to need no interpretation itself through arithmetical processes. When it does need such interpretation, it fails of its purpose as a quick, clear, convenient, and reliable picture.

Finally we note that the foregoing correctives and safeguards necessary for using the "difference" method correctly are, in actual fact, almost invariably neglected. Practically no one uses Marshall's subtangent comparison; few even mentally measure the heights of points above the base line,—much less mentally reckon, from these heights, the percentage rates of change; few even notice whether the base line is inserted or omitted or whether there are different base lines for different curves whose inconsistency needs to be allowed for; growth axes are seldom constructed; few make any effort to shut their eyes to straightness or parallelism in order to avoid being misled into assuming uniformity or similarity of *percentage* changes; few, even, use a diagram as a table of figures.

Professor Marshall says of the ordinary method of curve-plotting: "Its defects are such that many statisticians seldom use it except for the purpose of popular exposition, and for this purpose, I must confess, it has great dangers."† A business

* "On the Graphic Method of Statistics," Jubilee Vol. of the Royal Statistical Society, 1885, pp. 251-60. To help translate the deceptive slope of different lines, such as AA' and BB' in Fig. 6 *d* or Fig. 7 *d*, we produce these lines so that they cut the base line. If, when so produced, they are found to cut the base line at the same point, as is the case in Fig. 6 *d*, where they both cut at Y , then we may know that they have the same percentage slope. If, as in Fig. 7 *d*, they do not cut the base line in the same point, the one which cuts the further off has the smaller percentage rate of increase and the rates of increase per annum of the two lines at A and B are inversely proportional to the distances at which they cut the base as measured from X . Thus if, in Fig. 7 *d*, XY is $14/20$ of XZ the percentage change at A is $20/14$ of that at B . Again, in Fig. 4 *d* the slopes at A and A' are equal, as $XY = X'Y'$. All of these results may be proved geometrically.

† "On the Graphic Method of Statistics," Journal of the Royal Statistical Society, 1885, p. 251.

man whose attention was recently called to the advantages of the ratio chart recharted his business statistics and was startled to discover how he had been misled by the ordinary method.

After such a severe, but, I believe, just, indictment against ordinary or "difference" plotting it may be asked whether, as usually employed, such plotting is not worse than useless. It is certainly true that ordinarily it misleads to some extent. I have sometimes found myself hesitating to use a curve in a publication, because, on the one hand, of the space it would take up if the base line were inserted and, on the other, of the mistaken impression it might make if the base line were omitted.

The best that can be said for the difference method is: it always shows whether there is an increase or decrease; it usually displays the grosser contrasts at a glance; the base or zero line gives a means, lacking in the ratio method, for plotting zeros, for comparing positive and negative quantities, and for seeing in a simple and self-evident comparison the vertical elevations of points in a curve above the base line.

WAYS OF UTILIZING DIRECTION IN THE RATIO CHART.

The eye reads a ratio chart more rapidly than a difference chart or a table of figures. We may recapitulate what most easily catches the eye as follows:

1. If we see a curve ascending, and nearly straight, we know that the statistical magnitude it represents is increasing at a nearly uniform rate.

2. If the curve is descending, and nearly straight, the statistical magnitude is decreasing at a nearly uniform rate.

3. If the curve bends upward the rate of growth is increasing.

4. If downward, decreasing.

5. If the direction of the curve in one portion is the same as in some other portion it indicates the same percentage rate of change in both.

6. If the curve is steeper in one portion than in another portion it indicates a more rapid rate of change in the former than in the latter.

7. If two curves on the same ratio chart run parallel they represent equal percentage rates of change.

8. If one is steeper than another the first is changing at a faster percentage rate than the second.

9. The imaginary straight line most nearly representing, to the eye, the general trend of the curve, is its "growth axis," and represents the average rate of increase (or decrease); and the deviations of the curve from this growth axis are plainly evident without recharting.

10. The slope of the imaginary line between any two points on a curve indicates the average rate of change between the two.

WAYS OF UTILIZING ELEVATION IN THE RATIO CHART.

The preceding relates to *direction*. As to *elevation*, the eye can, with a little familiarity, translate vertical elevation into numerical ratio; for, as we have seen, a certain elevation represents a 10 per cent. increase, another a 100 per cent. increase, or doubling, etc., etc.*

To one accustomed to using the difference chart the use of elevations on the ratio chart may at first be confusing. But only a few minutes are necessary to learn its use; and this use of the mere elevation for measuring the ratios, or vertical distances, between any two points between the two magnitudes represented by those points, is really easier and more exact than the more self-evident method, in the ordinary chart, of comparing their two vertical distances from the base line.†

*For roughly interpreting elevations without having to read the marginal figures or measure distances, the original system of equidistant horizontals, as shown in Fig. 1 r, is not inconvenient. The chart is divided into horizontal 10 per cent. bands and the eye can approximately reckon how far any point on a statistical curve is vertically above any other point,—whether one band's breadth, or two, or any fractional part, and, therefore, whether it represents a magnitude 10 per cent. more, or 10 per cent. more than 10 per cent. more, i. e. 21 per cent. more, etc.

†We may also, if we wish, make much more exact comparisons than those afforded by the glance of an eye. We may exactly measure any slope such as that of a tangent to a curve at any point or such as that of a line connecting any two points on the curve. This may be done in accordance with the constructions of the radiating lines in Fig. 3 r; by drawing a line parallel to that of which the slope is desired, from any convenient point in the horizontal line "10". Let this cut the vertical line one unit to the right of said point; the height of this intersection above (or below) line "10" measures exactly the rate of ascent (or descent) of the slope to be measured.

Again we may exactly measure any ratio comparison between two points on the same curve—or on different curves for that matter. All we need to know is the vertical distance between the two points and compare this with the vertical scale above (or below) the line "10," as illustrated in Fig. 3 r. Such a comparison may be made by a draughtsman's dividers, parallel rulers, sliding triangle, or even by using the edge of a sheet of paper and a lead pencil. Thus, if the elevation, as measured, is found equal to the distance between "1" and "2" (or "10" and "20" or "100" and "200") then the upper point represents a statistical magnitude just twice that represented by the lower point; or again, if the measured elevation is equal to the distance between "1" and "1.37" (or "10" and "13.7" or "100" and "137") on the scale, then the upper point represents a statistical magnitude 37 per cent. greater than that represented by the lower point.

SUMMARY.

In the ratio method, then, a straight line* always represents a constant percentage rate of increase or decrease and, conversely, a constant percentage rate of increase or decrease is always represented by a straight line; a curve deviating from a straight line invariably implies that the percentage of change deviates correspondingly from constancy; any two curves or two portions of the same curve which are parallel represent exactly equal percentage rates of change; any two curves or portions of curves which show a contrast of direction always indicate a corresponding contrast in percentage change; if the numbers plotted are halved or changed in any other ratio, the resulting curve will simply be raised or lowered but will maintain exactly the same series of directions and therefore present the same appearance to the eye; if the scale is properly selected, a curve is never nearly horizontal except when it actually represents an almost infinitesimal rate of increase or decrease, nor is it ever nearly vertical except when it actually represents a rate correspondingly enormous; as there is no zero line there is no waste space on its account and the diagrams can be cut off close, both above and below the curve; there can be no juggling with base lines or scales; there is no need of special supplementary geometric constructions, such as Marshall's subtangent construction; there is no need of laborious calculations to reduce original figures to index numbers or percentages; there is no need of eliminating the growth axis (which, in the ratio method, is simply a straight line, the percentage deviations from which are apparent without special calculation or replotting).

The features of a curve which, whether we will or not, most "catch" the eye are concerned with comparative *direction*,—straightness or curvedness; steepness or flatness; parallelism or divergence. These features therefore ought to be, not

*It is interesting to note that engineers have found it advantageous to devise special plotting charts which will reduce parabolic, hyperbolic and probability curves of the ordinary charts to straight lines on the special charts, *e. g.*, see A. S. Langsdorf, "Methods for Determining the Equations of Experimental Curves," *Journal of the Association of Engineering Societies*, June, 1904, pp. 325-43; L. F. Harza, "Notes on Determination of Experimental Equations," *Wisconsin Engineer*, December, 1908; George C. Whipple, "Element of Chance in Sanitation," *Journal of the Franklin Institute*, Vol. 182, July 7, 1916, p. 37; Allen Hazen, "Storage to be Provided in Impounding Reservoirs for Municipal Water Supply," *Proceedings of the American Society of Civil Engineers*, Nov., 1913.

snares nor stumbling blocks, as they are in the "difference" chart, but aids or sign boards as they are in the "ratio" chart.

And, besides the full utilization of *direction*, we have, in the ratio chart, that of *elevation*. While the interpretation by the eye and mind of elevation requires a little preliminary training it soon becomes easier, more rapid and more accurate than the corresponding procedure for difference charts.

In a word, the ratio chart simply utilizes the natural powers of the eye. Consequently, when one is once accustomed to it, it never misleads, but always pictures a multitude of ratio relations at a glance, with absolute fidelity and without the annoyance of reservations or corrections.

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Professor Jevons seems to have been the first* to employ the idea of the ratio chart. He did not use prepared ruling but employed a table of logarithms and plotted the logarithms of the statistical numbers. While this procedure amounts to the same thing it fails to interpret the method to the average man, for it seems to lose touch with the actual statistical figures.

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*"During the years" (*viz.*, 1863 to 1884) "which have elapsed since this paper and previous essays on the value of gold were printed, I have not happened to learn that logarithmic diagrams had been used before their employment in these inquiries. It must be evident, however, that they furnish the true mode of representing all statistical and other numbers of which the ratios, not the absolute amounts, are in question. Now this is, strictly speaking, the case with almost all numbers used for statistical research, as apart from mere practical accounts." W. S. Jevons, "Investigations in Currency and Finance," London, (Macmillan), 1884, p. 128.

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